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DETONATION WAVES IN POLYDISPERSE GAS SUSPENSIONS OF UNITARY FUEL HAVING A CONTINUOUS PARTICLE SIZE DISTRIBUTION FUNCTION

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The first theoretical research on the wave dynamics of polydisperse gas suspensions was evidently carried out by Williams [1], who obtained intogrodifferential equations for the steady-state motion of a reactive collisionless mixture of a gaseous oxidizer and a sprayed liquid fuel with continuous distribution functions of drop size and velocity. Specific numerical calculations of the structure of a detonation wave in a gas–drop medium, however, were carried out in [1] using a distribution function that presumes a finite number of fractions of liquid particles. The laws of propagation of detonation waves in polydisperse sprays of fuel drops was subsequently studied within the framework of such a (multifractional) description of a collisionless disperse mixture (see, e.g., [2, 3]).

A system of integrodifferential equations for the unsteady motion of a multifractional, inert, collisional gas suspension with a particle velocity distribution function was used in [4] to calculate the interaction of a shock wave with a bidisperse cloud of dusty gas. Very weak shocks in a cloud of a polydisperse, inert, collisionless gas suspension with a continuous particle size distribution function were studied theoretically in [5]. The equations of unsteady motion of a polydisperse gas suspension with a continuous particle size distribution function in a linear (acoustic) approximation were generalized in [6] to the case of phase transitions in vapor–drop media. The burning of mixtures of gaseous oxidizer with metallic inclusions in a chemical reactor was investigated in [7] within the framework of the equations of a polydisperse, reactive gas suspension with a continuous distribution function of current particle sizes.

A model of the motion of a polydisperse, inert, collisionless gas suspension with a continuous particle size spectrum was developed in [8] and the adequacy of the proposed system of integrodifferential equations for describing actual shock processes in dusty gases was demonstrated, as confirmed by the satisfactory agreement between calculated and experimental data. In the present work we further develop the model of [8] for the case of a reactive, collisional gas suspension of unitary fuel with a continuous initial particle size distribution function.

1. Main Assumptions and Equations. Suppose that we have a polydisperse gas suspension of unitary fuel with a continuous particle size spectrum. The usual assumptions in the mechanics of continuous disperse media [8, 9] are used to describe the motion of such a mixture: The characteristic sizes of the particles and the distances between them are considerably less than the characteristic scale of variation of the macroscopic parameters of the flow; the effects of viscosity and heat conduction are important only in the interaction between phases; the mixture is rarefied and the particles are spherical; the particles do not fragment or merge in their collisions with each other; collisions between polydisperse particles are absolutely elastic; the energy of random motion of the polydisperse particles is much less than the energy of their translational motion; the influence of unsteady Basse and Archimedes forces and the associated masses on the parameters of the motion is negligible; in the undisturbed state the polydisperse gas suspension is spatially homogeneous in its disperse composition.

In addition, in accordance with [10, 11], we make the following assumptions about the mechanism of ignition and burning of particles of unitary fuel: particles are ignited when they are heated to the dissociation (gasification) temperature, with the temperature distribution inside the particles being uniform; during burning of the particles, the simplest chemical reaction of the type C → G occurs, where C and G are the chemical symbols for the condensed unitary fuel and for its gaseous combustion products; the temperature of the burning particles does not change, i.e., all of the heat reaching their surface goes into gasification of the fuel; the gaseous phase consists of two components – inert gas and combustion products of the particles of unitary fuel.

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During the phase transitions, the particle radius becomes a variable quantity. In this connection, for the index of a particle fraction we can, in accordance with [6, 12], take the initial radius $\hat{a}$ or the current radius a, which can be treated as the Lagrangian and Eulerian coordinates, respectively, of the given fraction in the space of particle sizes. Since there is a one-to-one relationship between the radii a and $\hat{a}$,

$$a = a(\hat{a}, x, t), \tag{1.1}$$

the particles at the space–time point (x, t), which at the time $t = 0$ had a radius in the interval $[\hat{a}, \hat{a} + d\hat{a}]$, will have a radius in the interval $[a, a + da]$, i.e., we have the equality

$$d\tilde{n} = \tilde{N}(a, x, t)\, da = d\hat{n} = \hat{N}(\hat{a}, x, t)\, d\hat{a}. \tag{1.2}$$

Here $d\tilde{n}$ and $\tilde{N}(a, x, t)$ are the number of particles per unit volume of mixture with a current radius in the interval from a to $a + da$ and their current size distribution function, respectively; $d\hat{n}$ is the number density of particles with initial radii in the interval from $\hat{a}$ to $\hat{a} + d\hat{a}$; $\hat{N}(\hat{a}, x, t)$ is the initial particle size distribution function.

The total number of particles of all sizes per unit volume of the mixture is

$$n = \int\limits_{\hat{a}_{\min}}^{\hat{a}_{\max}} \hat{N}(\hat{a}, x, t)\, d\hat{a} = \int\limits_{a_{\min}}^{a_{\max}} \tilde{N}(a, x, t)\, da$$

($\hat{a}_{\min}$ and $\hat{a}_{\max}$ are the minimum and maximum initial particle radii and $a_{\min}$ and $a_{\max}$ are the analogous current particle radii.

From (1.2) it follows that the functions $\tilde{N}$ and $\hat{N}$ are related by the simple equation

$$\tilde{N} = \hat{N} I^{-1} \quad \left(I = \frac{\partial a(\hat{a}, x, t)}{\partial \hat{a}}\Big|_{x,t}\right) \tag{1.3}$$

(I is the Jacobian of the transition from Lagrangian variables $\hat{a}$ to Eulerian variables a). The function (1.1) expresses the "law of motion" of a particle in dimensional space. When the particle radii are constant ($a = \hat{a}$), in particular, then $I = 1$ and the functions $\tilde{N}$ and $\hat{N}$ coincide identically in accordance with (1.3) ($\tilde{N} \equiv \hat{N}$). In general, $\tilde{N}$ and $\hat{N}$ differ. Descriptions of the motion of a polydisperse gas suspension using the functions $\tilde{N}$ and $\hat{N}$ are equivalent, but the equations of motion of the phases will differ. Such a difference can be demonstrated using the example of the equation of conservation of the number of particles.

For the fraction of particles with initial radii in the interval from $\hat{a}$ to $\hat{a} + d\hat{a}$, under the condition that the number $d\hat{n}$ is large enough for their motion to be described as that of a continuous medium, the equation of conservation of particle number is written in the form

$$\frac{\partial (d\hat{n})}{\partial t} + \frac{\partial (d\hat{n})\tilde{v}_2}{\partial x} = 0 \qquad \left(\tilde{v}_2 = \tilde{v}_2(a, x, t)\right), \tag{1.4}$$

where $\tilde{v}_2$ is the mass-average velocity of particles of radius a. Substituting Eq. (1.3) into (1.4), we have

$$\frac{\partial \hat{N} d\hat{a}}{\partial t}\Big|_{\hat{a}, x} + \frac{\partial \hat{N} \tilde{v}_2 d\hat{a}}{\partial x}\Big|_{\hat{a}, t} = 0.$$

Because $d\hat{a}$ is independent of the x and t variables, it can be taken out of the differential. The result is an equation representing the law of conservation of the number of particles in the polydisperse, reactive gas suspension:

$$\frac{\partial \hat{N}}{\partial t} + \frac{\partial \hat{N} \tilde{v}_2}{\partial x} = 0. \tag{1.5}$$

In the Eulerian variables a, the law of conservation of particle number takes the form [7]

$$\frac{\partial \tilde{N}}{\partial t} + \frac{\partial \tilde{N} \tilde{v}_2}{\partial x} + \frac{\partial \tilde{N} \tilde{v}_a}{\partial a} = 0 \qquad \left(\tilde{v}_a = \frac{d_2 a}{dt} = \frac{\partial a}{\partial t} + \tilde{v}_2 \frac{\partial a}{\partial x}\right) \tag{1.6}$$

($\tilde{v}_a$ is the linear rate of change of particle radius).

Equation (1.5) of conservation of particle number, written in the Lagrangian variables $\hat{a}$, is simpler to solve numerically than the analogous Eq. (1.6). In this connection, below we use only the variable $\hat{a}$.

Within the framework of the main assumptions given above, the system of equations for the plane–one-dimensional, unsteady motion of a polydisperse gas suspension of unitary fuel with a continuous initial particle size distribution function has the form [12]

$$\frac{\partial \rho_{1k}}{\partial t} + \frac{\partial \rho_{1k} v_1}{\partial x} = (k-1)\,J \quad (k = 1,2,\ \ \rho_1 = \rho_{11} + \rho_{12}),$$

$$\frac{\partial \rho_1 v_1}{\partial t} + \frac{\partial \rho_1 v_1^2}{\partial x} + \frac{\partial p}{\partial x} = -F, \quad \frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial \tilde{\rho}\,\tilde{v}_2}{\partial x} = -\hat{N}\tilde{j}, \quad \frac{\partial \hat{N}}{\partial t} + \frac{\partial \hat{N}\tilde{v}_2}{\partial x} = 0,$$

$$\frac{\partial \tilde{\rho}\,\tilde{v}_2}{\partial t} + \frac{\partial \tilde{\rho}\,\tilde{v}_2^2}{\partial x} = \hat{N}(\tilde{f} + \tilde{f}^{(c)}), \quad \frac{\partial \tilde{\rho}\,\tilde{e}_2}{\partial t} + \frac{\partial \tilde{\rho}\,\tilde{v}_2\tilde{e}_2}{\partial x} = \hat{N}\,(\tilde{q} - \tilde{j}\tilde{e}_2),$$

$$\frac{\partial}{\partial t}(\rho_1 E_1 + E_\rho) + \frac{\partial}{\partial x}(\rho_1 E_1 v_1 + E_{\rho v} + p\,(\alpha_1 v_1 + \alpha_v)) = 0,$$

$$F = \int\limits_{\Delta} \hat{N}\tilde{f}da \quad \Big(\int\limits_{\Delta} \hat{N}\tilde{f}^{(c)}d\hat{a} = 0\Big), \quad J = \int\limits_{\Delta} \hat{N}\tilde{j}d\hat{a}, \quad \rho_2 = \int\limits_{\Delta} \tilde{\rho}\,d\hat{a},$$

$$E_\rho = \int\limits_{\Delta} \tilde{\rho}\,\tilde{E}_2\,d\hat{a}, \quad E_{\rho v} = \int\limits_{\Delta} \tilde{\rho}\,\tilde{E}_2\tilde{v}_2\,d\hat{a}, \quad \alpha_v = \int\limits_{\Delta} \frac{\tilde{\rho}}{\rho_2^0}\tilde{v}_2\,d\hat{a}, \quad \int\limits_{\Delta} \equiv \int\limits_{\hat{a}_{\min}}^{\hat{a}_{\max}},$$

$$\rho_{1k} = \rho_{1k}^0\,\alpha_1, \quad \rho_i = \rho_i^0\,\alpha_i \quad (i,\, k = 1,\, 2), \quad \alpha_1 + \alpha_2 = 1,$$

$$\tilde{\rho} = \tilde{m}_2\hat{N}(\hat{a}, x, t), \quad E_1 = e_1 + 0{,}5\,v_1^2, \quad \tilde{E}_2 = \tilde{e}_2 + 0{,}5\,\tilde{v}_2^2,$$

$$\tilde{f} = \tilde{f}_\mu - \tilde{j}\tilde{v}_2, \quad \tilde{m}_2 = \frac{4}{3}\pi a^3\rho_2^0, \quad a = a(\hat{a}, x, t).$$

Here we give the laws of conservation of the masses of inert gas ($k = 1$) and gaseous products of the chemical reaction ($k = 2$), conservation of momentum for the gaseous phase as a whole, and conservation of mass, number density, and momentum of an individual particle fraction, as well as the equation of heat influx to particles of one fraction and the law of conservation of the total energy of the entire mixture; parameters of the gaseous phase are marked by the subscript 1, parameters of the disperse phase as a whole by 2, and parameters of the particle fraction with a radius $a = a(\hat{a}, x, t)$ by a tilde; ρ, ρ^0, α, v, e, and E are the average and true density, volume content, mass velocity, specific internal energy, and total energy of the components of the gas or gaseous mixture, as well as the particle fractions or the disperse phase as a whole; p is the pressure of the gaseous mixture; F and J are the intensities of the force interaction and of mass transfer between the gaseous phase and the disperse phase as a whole per unit volume of the mixture; $\tilde{f}_\mu$ is the force of aerodynamic action by the gaseous phase on an individual particle of radius a; $\tilde{f}^{(c)}$ is the force exerted on a particle of radius a in collisions with particles of other sizes; $\tilde{m}_2$ is the mass of a particle of radius a; $\tilde{q}$ and $\tilde{j}$ are the intensities of heat and mass transfer between an individual particle of radius a and the gaseous phase; E_ρ is the total energy of the disperse phase per unit volume of the mixture; $E_{\rho v}$ is the total energy flux of particles of all sizes through a unit area per unit time; α_v is the volume-average velocity of the disperse particles.

For the equations of state of the gaseous components we take the equations of state of an ideal, calorically perfect gas. For the gaseous carrier phase as a whole, we assume the condition of additivity of the internal energy of the mixture with respect to the masses of its components to be valid, as is Dalton's law [12]:

$$p_{1k} = \rho_{1k}^0 R_{1k} T_1, \quad e_{1k} = c_{1k}(T_1 - T_0) + e_{1k}^0 \quad (k = 1,\, 2),$$

$$p = \sum_{k=1}^{2} p_{1k}, \quad e_1^0 = \sum_{k=1}^{2} x_{1k} e_1^0, \quad c_1 = \sum_{k=1}^{2} x_{1k} c_{1k},$$

$$e_1 = c_1(T_1 - T_0) + e_1{}^0, \quad x_{1k} = \frac{\rho_{1k}}{\rho_1} \quad (R_{1k},\, c_{1k},\, e_{1k}^0,\, T_0 = \text{const}).$$

Here and later p_{1k}, R_{1k}, c_{1k}, and e_{1k} are the partial pressure, gas constant, specific heat at constant volume, and specific internal energy of the kth gaseous component; c_1 and e_1^0 are the specific heat of the gas as a whole at constant volume and the internal energy of the gaseous mixture at the temperature T_0; T_1 is the current gas temperature.

For the incompressible particles of unitary fuel we take the equation of state [12]

$$\rho_2^0 = \text{const}, \quad \tilde{e}_2 = c_2(T_2 - T_0) + e_2^0 \quad (e_2^0,\ c_2 \equiv \text{const}),$$

where c_2 and $\tilde{T}_2$ are the specific heat and temperature of particles of radius a; e_2 and e_2^0 are the specific internal energies of the particles at the temperatures $\tilde{T}_2$ and T_0.

The constant e_{11}^0 for the inert component of the gas mixture can be arbitrary, while the constants e_{12}^0 and e_2^0 for the reactive components of the disperse medium must satisfy the following normalization condition [12]:

$$e_2^0 - e_{12}^0 = Q^0 - \left(\frac{p_0}{\rho_2{}^0} - \frac{p_{120}}{\rho_{120}^0}\right).$$

Here Q^0 is the calorific value of the unitary fuel; p_{120} and ρ_{120}^0 are the partial pressure and true density of the gaseous combustion products, which are part of the gaseous phase and are at a temperature T_0 and a pressure p_0.

Equations for the force $\tilde{f}_\mu$ of viscous friction and the force $\tilde{f}^{(c)}$ of collisions between polydisperse particles are written in the form [8]

$$\tilde{f}_\mu = \frac{1}{2}\pi a^2 \tilde{C}_d \rho_1^0 |v_1 - \tilde{v}_2|(v_1 - \tilde{v}_2),$$

$$\tilde{f}^{(c)} = \frac{8}{3}\pi^2 \rho_2^0 \int\limits_{\Delta} æ^F f(a, a_1)\, \hat{N}(\hat{a}_1, x, t)\, |\tilde{w}|\, \tilde{w}\, d\hat{a}_1,$$

$$\tilde{C}_d = \frac{24}{\widetilde{\mathrm{Re}}} + \frac{4}{\sqrt{\widetilde{\mathrm{Re}}}} + 0{,}4, \quad \widetilde{\mathrm{Re}} = \frac{2a\rho_1^0|v_1 - \tilde{v}_2|}{\mu_1},$$

$$f(a, a_1) = \frac{(aa_1)^3(a + a_1)^2}{(a^3 + a_1^3)}, \quad \mu_1 = x_{11}\mu_{11} + x_{12}\mu_{12},$$

$$\tilde{w} = \tilde{v}_2(a, x, t) - \tilde{v}_2(a_1, x, t), \quad a_1 = a_1(\hat{a}_1, x, t),$$

where $\tilde{C}_d$ and $\widetilde{\mathrm{Re}}$ are the aerodynamic drag coefficient and the Reynolds number for a single spherical particle moving in a gas stream; μ_{1k} ($k = 1, 2$) and μ_1 are the dynamic viscosities of the k-th component of the gas mixture and of the carrier phase as a whole; $æ^{(F)}$ is a coefficient that allows for the fraction of momentum transferred on the average from a particle of radius a to a particle of radius a_1 in one collision. According to the recommendation of [9], based on extrapolation of the experimental data of [13], for relative velocities of colliding particles $|\tilde{w}|$ of ~10 and ~100 m/sec, $æ^{(F)}$ is ~0.1 and ~0.01, respectively.

The expressions for the intensities of contact heat transfer and interphase heat transfer have the form [12]

$$\tilde{q} = 2\pi\widetilde{\mathrm{Nu}}\lambda_1(T_1 - \tilde{T}_2)\, H(-\tilde{j}), \quad \tilde{j} = 4\pi a^2 \rho_2^0 v_S \left(\frac{p}{p_S}\right)^{\varphi} H(T_2 - T_S),$$

$$\widetilde{\mathrm{Nu}} = 2 + 0{,}6\widetilde{\mathrm{Re}}^{1/2}\mathrm{Pr}^{1/3}, \quad \mathrm{Pr} = \frac{c_{p1}\mu_1}{\lambda_1};$$

$$\lambda_1 = \sum_{k=1}^{2} x_{1k}\lambda_{1k}, \quad c_{p1} = \sum_{k=1}^{2} x_{1k}c_{p1k} \quad (T_S, v_S, \varphi, p_S \equiv \text{const}).$$

Here $\widetilde{\mathrm{Nu}}$ and Pr are the Nusselt and Prandtl numbers; c_{p1k}, λ_{1k}, c_{p1}, and λ_1 are the specific heats at constant pressure and the coefficients of thermal conductivity of the gas components and of the gaseous phase as a whole; T_S is the ignition temperature of the unitary fuel; $H(z)$ is a Heaviside function of the argument $z = -\tilde{j}$, $\tilde{T}_2 - T_S$.

2. Statement of the Problem and Some Results of the Numerical Investigation. Let a homogeneous cloud of a polydisperse gas suspension of unitary fuel with a continuous particle size distribution function be in the region ($x_* \le x < \infty$). A plane shock wave with a triangular profile of mass gas velocity, simulating an explosive wave in the zone ($0 < x \le x_f$,

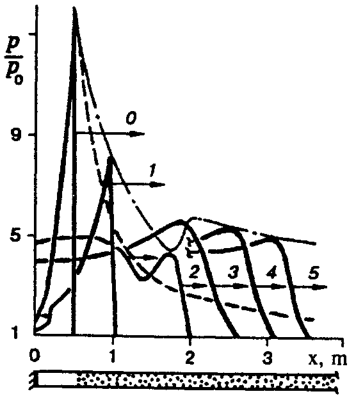

Fig. 1

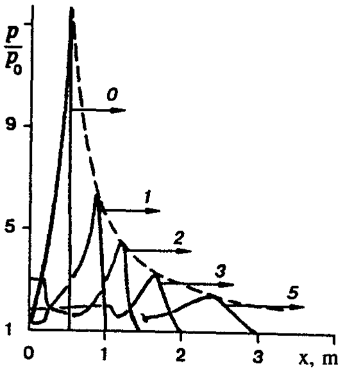

Fig. 2

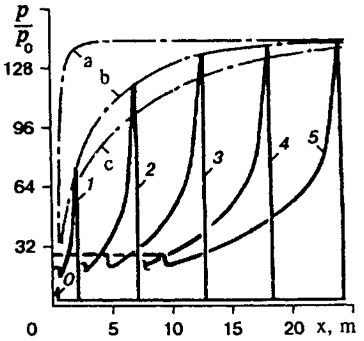

Fig. 3

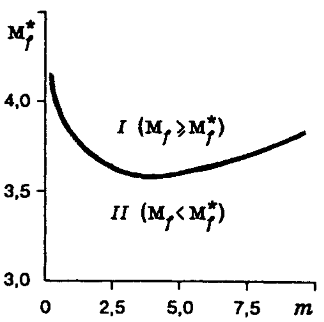

Fig. 4

$x_f < x_*$), strikes this cloud from the region of inert gas ($0 \leq x < x_*$) at the initial time $t = 0$. The purpose of the investigation is to study the processes of combustion and detonation arising in the gas suspension under the action of the shock.

The shock's intensity is given by the Mach number of its leading front, $M_f = D_f/a_{110}$, where D_f is the shock's velocity in the laboratory coordinate system and a_{110} is the adiabatic speed of sound ahead of the shock discontinuity. The gas parameters at the shock discontinuity, as well as those marked by an f subscript, are expressed in terms of M_f on the basis of the Rankine–Hugoniot equations:

$$\frac{v_{1f}}{a_{110}} = \frac{2}{(\gamma_{11}+1)}\left(M_f - \frac{1}{M_f}\right), \quad \frac{\rho^0_{1f}}{\rho^0_{10}} = \frac{(\gamma_{11}+1)M_f^2}{2+(\gamma_{11}+1)M_f^2},$$
$$\frac{p_f}{p_0} = 1 + \frac{2\gamma_{11}}{\gamma_{11}+1}(M_f^2 - 1), \quad \frac{T_{1f}}{T_0} = \left(\frac{p_f}{p_0}\right)\left(\frac{\rho^0_{1f}}{\rho^0_{10}}\right)^{-1}. \tag{2.1}$$

The gas parameters outside the shock discontinuity, in the rarefaction zone, are given on the basis of isentropic functions for a simple wave with a triangular gas velocity profile [14]:

$$v_1 = v_{1f}\frac{x}{x_f}, \quad p = p_f\sigma^{\gamma_{11}}, \quad \rho_1{}^0 = \rho^0_{1f}\sigma, \quad \rho^0_{11} = \rho^0_1, \quad \rho_{12} = 0, \quad \alpha_1 = 1, \quad T_1 = \frac{p}{\rho^0_1}$$
$$\left(\sigma = \left[1 - \frac{(\gamma_{11}-1)}{2}\frac{v_{1f}}{a_{1f}}\left(1 - \frac{x}{x_f}\right)\right]^{\frac{2}{(\gamma_{11}-1)}}, \; a^2_{1f} = \gamma_{11}\frac{p_f}{p_0}\right) \; (0 < x \leq x_f). \tag{2.2}$$

In the undisturbed region beyond the limits of the shock the parameters of the phases have the following distributions:

$$v_1 = 0, \quad p = p_0, \quad \rho^0_1 = \rho^0_{110}, \quad T_1 = T_0, \quad \alpha_2 = \alpha_{20}H(x - x_*),$$
$$\rho_{12} = 0, \quad \alpha_1 = 1 - \alpha_2, \quad \rho_1 = \rho^0_1\alpha_1,$$
$$\hat{v}_2(\hat{a}, x, 0) = 0, \quad \tilde{T}_2(\hat{a}, x, t) = T_0, \quad a(\hat{a}, x, 0) = \hat{a},$$

$$\hat{N}(\hat{a}, x, 0) = \hat{N}_0(\hat{a}) \quad (x_f < x < \infty).$$

For distributions of the disturbed parameters of the gaseous phase given in the form (2.1), (2.2), the initial energy of the initiating explosion is found from the integral

$$E_0 = \int_0^{x_f} \rho_1^0 \left(e_1 + \frac{v_1^2}{2}\right) dx. \tag{2.3}$$

At the left-hand boundary of the region, which is a rigid wall ($x = 0$), we set up the condition that the gas velocity vanish,

$$v_1(0, t) = 0,$$

and the condition of free flow through of particles,

$$\tilde{v}_2(0_-, t) = \tilde{v}_2(0_+, t).$$

No boundary condition was set up at the right-hand end of the calculation region, since perturbations of the parameters of the phases did not reach that boundary during the entire time of the investigated wave flow.

The problem was solved numerically by the method of large particles [14-16], with the integral equations being calculated by Simpson's method [17]. The accuracy of the calculations was monitored by a recalculation with reduced step sizes in time and space, as well as in the number of particle fractions.

The calculations were made for mixtures of air and particles of unitary fuel (powder of brand N). We used the following values of the thermodynamic parameters of the phases and their components [11]: for the inert gas–air system $\rho_{110}^0 = 1.29$ kg/m^3, $p_0 = 0.1$ MPa, $T_0 = 300$ K, $\gamma_{11} = 1.4$, $R_{11} = 286.69$ m^2/(sec^2·deg), $\mu_{11} = 1.85 \cdot 10^{-5}$ kg/(m·sec), $\lambda_{11} = 2.57 \cdot 10^{-2}$ kg·m/(sec^3·deg), and $a_{110} = 341$ m/sec; for the gaseous products of combustion of the powder $\rho_{120}^0 = 1.03$ kg/m^3, $T_0 = 300$ K, $\gamma_{12} = 1.3$, $R_{12} = 328$ m^2/(sec^2·deg), $\mu_{12} = 1.73 \cdot 10^{-5}$ kg/(m·sec), and $\lambda_{12} = 8.37 \cdot 10^{-2}$ kg·m/(sec^3·deg); for the model N powder $\rho_2^0 = 1550$ kg/m^3, $c_2 = 1466$ m^2/(sec^2·deg), $T_S = 473$ K, $\varphi = 0.7$, $v_S = 6.5 \cdot 10^{-4}$ m/sec, $p_S = 0.1$ MPa, and $Q^0 = 1.93$ MJ/kg. Here γ_{1k} are the adiabatic indices of the inert gas ($k = 1$) and the gaseous products of the chemical reaction ($k = 2$).

In accordance with [18], in which a histogram of the fractional composition of the gas suspension was determined experimentally, and by analogy with [8], we offer the following approximation of the experimental histogram by a normal–logarithmic law:

$$\hat{N}_0(\hat{a}) = \frac{k n_0 \hat{a}^{-1}}{\sqrt{2\pi} \ln\sigma} \exp\left[-\frac{1}{2}\left(\frac{\ln\hat{a} - M}{\ln\sigma}\right)^2\right],$$
$$n_0 = \int_\Delta \hat{N}_0(\hat{a})\, d\hat{a}, \quad k = n_0^{-1} \int_0^\infty \hat{N}_0(\hat{a})\, d\hat{a} \tag{2.4}$$

(M is the mathematical expectation and $\ln\sigma$ is the dispersion). For the particle spectrum measured in [18] ($\hat{a}_{\min} = 2.5$ μm and $\hat{a}_{\max} = 32.5$ μm), the fractional parameters of the particle size distribution (2.4) have the values [8] $M = 0.393$, $\ln^2\sigma = 2.869$, and $k = 1.214$.

In all of the calculated variants, the initial length of the initiating shock was $x_f = 0.47$ m and the coordinate of the left-hand boundary of the cloud was $x_* = 0.5$ m. The intensity of the initiating shock, given by the Mach number M_f, was varied in the range $0 < M_f \leq 5$. The initial energy of the explosion was also varied in the process, in accordance with (2.3). The initial relative mass content of the disperse phase in the mixture ($m = \rho_{20}/\rho_{10}$) was varied in the range $0 < m \lesssim 10$.

In Fig. 1 we give typical calculated pressure profiles of the gaseous phase occurring in a cloud of a polydisperse gas suspension of unitary fuel ($m = 6$) in the process of dynamic loading by a shock ($M_f = 3.5$). Curves *0-5* correspond to the times 0, 0.91, 3.66, 5.49, 7.33, and 9.16 msec. Arrows show the directions of propagation of the incident, transmitted, and reflected shocks. The envelopes of peak pressures behind the fronts of transmitted shocks in a reactive gas suspension ($J \neq 0$)

and an identical inert ($J = 0$) gas suspension are plotted with dash-dot and dashed lines. The pressure distributions in a model, chemically inert, polydisperse gas suspension ($J = 0$) corresponding to Fig. 1 are shown separately in Fig. 2.

As follows from these calculated data, a transmitted shock passing into the gas suspension and a compression wave reflected into the region of "pure" gas are produced in the interaction of an incident shock with a cloud of unitary fuel. In the initial stage of its motion, the transmitted shock is strongly damped owing to the comoving rarefaction wave and heat losses from the gas stream due to heating of particles of the unitary fuel. From the time of ignition of particles in the stream, a combustion wave is formed, moving in the direction of propagation of the leading shock. The compression waves generated in the zone of particle combustion in this case propagate downstream and decrease the rate of damping of the transmitted (leading) shock.

The slight strengthening of the combustion wave observed at ~4-5 msec occurs in the compression wave reflected from the wall, which, propagating through the disturbed gas suspension, always outruns the leading shock. The existing dominance of damping of the shock over amplification of the combustion wave, however, causes the particle ignition front to gradually lag behind the leading shock front and results in a regime of fading combustion of the polydisperse gas suspension. The latter indicates that the intensity of the initiating shock ($M_f = 3.5$) and hence the initial explosive energy ($E_0 = 1.64$ MJ/m^2) are insufficient to achieve detonation in a powder gas suspension with $m = 5$.

In Fig. 3 we give an example of a calculation of the dynamic action of a high-intensity shock ($M_f = 7$) on a cloud of a polydisperse gas suspension of unitary fuel ($m = 5$); gas pressure profiles at the times 0, 1.83, 3.66, 5.49, 7.33, and 9.16 msec are shown (curves *0-5*); the envelopes of peak pressures behind transmitted waves in a polydisperse medium with $\hat{a}_{\min} = 2.5$ μm and $\hat{a}_{\max} = 32.5$ μm (line b) and in a monodisperse mixture with $\hat{a}_* = 2.5$ and 32.5 μm (lines a and c, respectively) are plotted with dash–dot lines.

As seen from Fig. 3, the intensity of the initiating shock ($M_f = 7$) and the initial explosive energy ($E_0 = 9.32$ MJ/m^2) are sufficient to achieve a regime of detonative combustion of the polydisperse gas suspension of unitary fuel. In this case, the shock passing through the gas suspension dies out only until a combustion wave has formed in the stream. From the time of ignition of particles in the mixture, owing to the release into the gas of the hot gaseous products of the chemical reaction, one observes gradual strengthening of the combustion wave and the shock. As these waves become stronger, their propagation velocities increase and the difference in wave velocities decreases. In the limit, at sufficiently long times, the velocity of the combustion wave asymptotically approaches the velocity of the leading shock and a standing detonation wave is formed in the gas suspension. The characteristic distance at which the combustion wave in the polydisperse mixture arrives at the steady detonation regime is ~25 m. This predetonation distance for a polydisperse mixture lies between the analogous distances for monodisperse gas suspensions with particle sizes $\hat{a} = \hat{a}_{\min}$ and $\hat{a} = \hat{a}_{\max}$.

On the basis of the foregoing, the question arises of which of these two regimes of combustion of a reactive gas suspension occurs under given initial conditions of the problem. The main parameters characterizing the initial conditions are the Mach number M_f and the length x_f of the initiating shock, the relative mass content m of particles in the mixture, and the particle size distribution function $\hat{N}_0(\hat{a})$ for a polydisperse gas suspension or the particle radius $\hat{a}_*$ for a monodisperse mixture.

For each function $\hat{N}_0(\hat{a})$, the set of values of the parameters M_f, x_f, and m can be divided into two domains, in the first of which, corresponding to the regime of damped combustion, the condition

$$F(M_f, x_f, m) \leq 0 \tag{2.5}$$

is satisfied, while in the second domain, corresponding to the regime of heterogeneous detonation, we have

$$F(M_f, x_f, m) > 0 \tag{2.6}$$

[$F(M_f, x_f, m)$ is some function]. The values of the parameters M_f, x_f, and m (belonging to the boundary between the two domains) for which $F(M_f, x_f, m) = 0$ will be called critical values.

Solving the inequalities (2.5) and (2.6) for M_f, for the regime of damped combustion of a gas suspension we obtain the condition

$$M_f \leq M_f^*(m, x_f),$$

while for the detonative combustion regime we obtain the condition

$$M_f > M_f^*(m, x_f)$$

(M_f^* is the critical Mach number of the initiating shock).

In Fig. 4 we show the calculated behavior of the critical (minimum) Mach number $M_f^*(m, x_f)$ of the initiating shock for $x_f = 0.47$ m and $0.1 \leq m \leq 10$ for a polydisperse gas suspension of unitary fuel having a normal-logarithmic particle size distribution (2.4) with $\hat{a}_{\min} = 2.5$ μm and $\hat{a}_{\max} = 32.5$ μm. The numbers I and II in the (M_f^*-m) plane denote the domains of gas suspensions that can ($M_f \geq M_f^*$) and cannot be detonated ($M_f < M_f^*$) under the conditions of their shock loading.

The critical Mach number of the initiating shock is seen to vary nonmonotonically as a function of the initial relative mass content m of fuel. In accordance with the given function, M_f^* decreases as the initial mass content of fuel increases from 0.1 to some value $m = m_*$ ($m_* \approx 4$), while it increases at higher mass contents ($m_* \leq m \leq 10$). The nonmonotonic variation of M_f^* as a function of m can be explained briefly by the fact that the fractions of the energy of the initiating shock (E_0, $\partial E_0/\partial M_f > 0$) expended on particle acceleration (E_0^v) and heating (E_0^T) depend in opposite ways on the initial concentration of the disperse phase, i.e., E_0^v decreases and E_0^T increases with increasing m. Specific expressions for E_0^v and E_0^T are not given in the present paper because they are cumbersome.

There is great interest in the question of the adequacy of the description of combustion and detonation in reactive, polydisperse gas suspensions within the framework of the model of a monodisperse mixture containing inclusions with a certain effective radius a_*, determined by the initial particle size distribution function [6]:

$$a_* = \hat{a}_{m,n} = \left[\frac{\int_\Delta \hat{N}_0(\hat{a})\hat{a}^m\,d\hat{a}}{\int_\Delta \hat{N}_0(\hat{a})\hat{a}^n\,d\hat{a}}\right]^{\frac{1}{m-n}}, \qquad \begin{array}{l} a_{m,n} = a_{n,m}, \\ m,\, n \equiv \text{const}, \\ m \neq n. \end{array}$$

Our calculations based on the model of a monodisperse mixture with $a_* = a_{6.5} = 23.5$ μm and $a_* = a_{3.0} = 15.5$ μm describe satisfactorily (with an 8% error) the results of calculations based on the model of a polydisperse gas suspension, shown in Figs. 3 and 4, respectively.

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